COSMICAL RELATIONS OF LIGHT TO GRAVITY.

By Pliny Earle Chase.

Prof. Kirkwood's very interesting presentation of the evidence which indicates special lines of disturbance on the Sun's surface, furnishes a new analogy to guide the researches of investigators. The well known dependence of one class of magnetic fluctuations on the position of ocean meridians, strengthens his hypothesis of similar meridians beneath the solar photosphere* which may possibly be detected by spectroscopic observations, while the coincidence of luminous, magnetic, and gravitating lines encourages renewed efforts to trace out the fundamental harmonics of our planetary system.

Wheatstone's experiments have been generally regarded as proving that the velocity of electricity is greater than that of light. But the outbreak of the solar spot recorded by Sir John Herschel, and the simultaneous agitation of the magnetic needles at Kew and elsewhere, render it probable that electrical action is sometimes, if not always, transmitted with precisely the same velocity as light. May it not be that the induction between the successive coils of a wire, however widely they may be separated, produces a spark before the electric current has traversed the whole extent of the wire? Or, if the wire were transparent, is there any reason for supposing that it would transmit a wave of light less rapidly than one of electricity?

The analogies to which attention has been called by numerous observers, between phenomena which are dependent upon various forms of force, may be supplemented by relations, no less curious and interesting, of light to cosmical gravitation, some of which are shown in the following equations. They appear to open a new field for inquiry, in which analysts may, perhaps, find profitable employment.

Let the sun and planets be denoted by the following subscript figures. ☉$_1$; ☿$_2$; ♀$_3$; ⊕$_4$; ♂$_5$; inner asteroid$_6$; mean asteroid$_7$; superior asteroid$_8$; ♃$_9$; ♄$_{10}$; ♅$_{11}$; ♆$_{12}$.

Let h be the modulus of solar light, on the hypothesis that the luminiferous æther is an indefinitely elastic, material medium, and that, therefore, $h = \frac{u^2}{2g_1}$, u denoting the velocity of light.

V = velocity, and T = time of theoretical planetary revolution at the surface of the sun, or of a planet.

v_n = velocity, and t_n = time, of axial rotation of n

r = radius

m = mass

d = mean distance from sun

γ = centre of gyration = $\sqrt{\frac{2}{5}}$

* See also Henshall on ♂ of ☿, ♀, and ♃ with same face of sun. *Cosmos.* xvii., Nov. 1860. p. 573.

$e = 1 +$ orbital excentricity

$i =$ *effective* inertia of votation $=$ moment of inertia divided by time of revolution $= m\sqrt{d}$ Then

1. $t_1 = \frac{2h}{u} = \left(\frac{2h}{g_1}\right)^{\frac{1}{2}} \therefore g_1 t_1 = u$, and $v = 2g_1 \frac{\pi r_1}{u} = \frac{1}{4} \cdot \frac{2g_1}{u} \cdot \frac{surface}{r}$

2. $\frac{d_7}{r_1} = \left(\frac{h}{r_1}\right)^{\frac{1}{2}} = \frac{i}{m}$ of æther

3. $\frac{d_{12}}{d_2} = \left(\frac{h}{r_1}\right)^{\frac{1}{3}}$

4. $d_4 = \frac{d_7}{\pi}$

5. $\frac{d_4}{\pi r_1} = \left(\frac{m_1}{m_4}\right)^{\frac{1}{3}}$

6. $d_2 = \left(\frac{\gamma}{e_4}\right)^2 d_4$

7. $d_3 - \frac{3\gamma}{e_4} d_2$

8. $d_5 = 2\,(d_4 — \gamma d_2)$

9. $\frac{d_9}{r_1} = \frac{m_1 \times e_9}{m_9}$

10. $d_{10} = \frac{d_{12}}{\pi}$

11. $\sqrt{2}\; d_6 = d_7$

12. $\gamma\, d_8 = d_6$

13. $\frac{d_{12} \times d_{11}}{d_4 \times r_1} = \left(\frac{m_1}{m_4}\right)^{\frac{1}{2}}$

14. $d_i\,({}_{2\cdot\,3\cdot\,4\cdot\,5\cdot}) = 2\,\gamma\, d_2$

15. $\frac{V_1}{v_1} = \frac{d_4}{r_1}$

16. $\frac{V_4}{v_4} = \frac{d_9 \times e_9}{r_4} \times \pi$

The motion of the air in the earth's annual revolution and daily rotation, is slightly undulatory, but hardly perceptibly differing from a regular ellipse. Its motions are controlled, mainly by solar, and subordinately by terrestrial force, the former giving a motion of 63.8, and a moment of inertia of 543,000,000 times the latter. According to Marriotte's law, the specific gravity of the atmosphere should be determined by the conjoined pressure of solar and terrestrial gravity. The liquid and solid portions of the earth, however, are not subject to Marriotte's Law.

In any fluid which is simultaneously affected by two attracting masses, e. g: by the earth and the sun, it would seem that two systems of waves should be generated, moving with velocities v, v^1 such that $v = \sqrt{2\,gh}$ $\quad v^1 = \sqrt{2\,g^1 h^1}$

But if the fluid is on the earth's surface, $h = h^1$, while at the centres of force, $g : g^1 :: m : m^1$.

In the orbital motion the pressure of solar force is nearly constant, but terrestrial gravity tends to maintain the atmosphere at a uniform level, or in a constant volume. Now the kinetic energy under constant pressure : that under constant volume : : 1.421 : 1, or very nearly : : $\sqrt{2}$: 1.

M. Trèves found that the number of oscillations in a tuning fork was increased $\frac{1}{817}$ by magnetizing the fork. Farther experiments are desirable to determine whether his result may be accepted as a general one, but it may be temporarily regarded as curiously coincident with our hypothetical case. in which

17. $v : v^1 : : \sqrt{2m} : \sqrt{m^1}$
$817 : 1 : : \sqrt{2m_1} : \sqrt{m_4}$

18. $\sqrt{2m_1} : \sqrt{m_4}$: : sp. gr. water : sp. gr. air (at mean temperature)

19. $\left(2m_1\right)^{\frac{1}{4}} : \left(m_4\right)^{\frac{1}{4}} : g_1 : g_4$

20. $2\, g_4\, t_4 \times \frac{g_4}{g_1} = V_4$

21. The inertia of the air which is retarded by the thermal and tidal "brakes" appears to be overcome and the wave-equilibrium restored, after g_4 has acted for a sufficient time to give V_4.

22. $d_4 = \left(\frac{\text{yearly}}{\text{daily}}\text{ barometric range}\right)^2$ at St. Helena $\times\ g_4\, t_4{}^2$

23. $2\sqrt{2m_1} \times (e_4)^2 : \sqrt{m_4} : : g_4 : g_1 \times \left(\frac{r_1}{d_4}\right)^2$

24. Mean vel. of sound = mean vel. of air.

25. If $\frac{d_9}{r_1}$ be divided in proportion to the i of the several planets, Jupiter's proportion will be $\frac{8}{13}$ of 1125.84 = 692.83, and 692.83 $\times\ V_1 = u$.

It may be desirable to modify some of these equations by considerations connected with centrifugal force. The closeness of the principal analogies may be illustrated by a few examples, in which I assume the following values as a basis of comparison : $\lambda \frac{g_1}{g_4} = 1.449662$; $\lambda\, r_1 = 5.630334$ ∴ $\lambda\, h =$ 11.302517 ;—and Newcomb's estimates, $\frac{m_1}{m_4} = 326{,}800$; $d_4 = 92{,}380{,}000$; u = 185,600.

		Spörer.	Faye.	Carrington.	Kirkwood.	Theory.	Mean.
1.	t_1 =	24.62447 dy.	25.07472 dy.	24.9711 dy.	24.82594 dy	25.0297 dy.	24.9052 dy.
	∴ u =	188,697 m.	185,267 m.	186,035 m.	187.123 m.	185,600 m.	186,528 m.

			Theoretical.	Assumed.
3.	$\frac{d_{12}}{d_2} = \frac{30.070552}{\cdot 3870984}$	∴	$\frac{h}{r_1} = 468{,}770$	465,604.
4.	$\frac{d_7}{r_1} = 682.3516$	∴	$\frac{d_7}{d_4} = 3.1416$	3.153.

5. $\frac{d_4}{\pi r_4} = 68.88$ $\therefore \frac{m_1}{m_4} = 326,800$ [327,280]*

10. $\frac{d_{12}}{d_{10}} = \frac{30.070552}{9.5388}$ $\therefore \frac{d_{12}}{d_{10}} = 3,1416$ 3,1524.

15. $\frac{d_4}{r_1} = 216.395$ $\therefore \frac{V_1}{v_1} = 216,395$ 218,142.

17. $\frac{v}{v^1} = 817$ $\therefore \frac{m_1}{m_4} = 333,750$ 326,800.

18. Mean s. g. $\frac{\text{water}}{\text{air}} = 807.45$† $\therefore \frac{m_1}{m_4}$ 325,380 326,800.

19. $(2 \times 326,800)^{\frac{1}{4}} = 28.4\dot{3}$ $\therefore \frac{g_1}{g_4} = 28.4\dot{3}$ 28.162.

20. $2 g_4\, t_4 \times \frac{g_4}{g_1} = 18.7$ m. $\therefore V_4 = 18.7$ 18.4.

21. $2 g_4 \times 50\frac{1}{4} \times 60 = 18.376$ m. $\therefore V_4 = 18.376$ 18.4.

22. $\left(\frac{.135}{.067}\right)^2 \times g_4\, t_4{}^2 = 92,361,900$ m. $\therefore d_4 = 92,361,900$ 92,380,000.

24. $\frac{\cos. 43° \times 2 \pi r_4}{t_4} = 1112.97$ ft. $\therefore$ vel. $= 1112.97$ 1118.09‡

25. $692.83\ V_1 = 187,750$ m. $\therefore u = 187,750$ 185,600.

26. The earth's proportion would be .000862 of 1125.84 = .97, and

$$.97 \times \frac{V_1}{v_1} = \frac{4\pi}{3} \times \frac{\text{mass of sun}}{\text{mass of planets}}$$

27. $.97 \times 2\, g_4\, T_4 = u.$

The following estimates of the sun's mass and distance, and the velocity of light, are derived from the foregoing equations :

	From magnetic acceleration.	From sp. gravity of air.	From length of day.	From atmospheric inertia.	From annual baro-m'ic range.	From inertia of Jupiter.	From mean estimate of $g_1 t^1$
Sun's mass	333,750	325,380	341,560	325.520	326,610	338,490	321,940
Sun's distance	93,033,200	92,246,000	93,886,300	92,260,000	92,361,900	93,450,000	91,920,000
Vel. of light	186,910	185,330	188,630	185,360	185,560	187,750	184,670

The study of gaseous molecular motions may, perhaps, be aided by the analogies of luminous vibrations. The equation $u = g_1\, t_1$ seems to be an important one. A solitary planet or particle would acquire the velocity of revolution in a circular orbit in $\frac{1}{6}$ of the time of *revolution*, but the particles of the hypothetical elastic fluid to which the luminous vibrations are attributed, under the combined pressure of g_1 and of their own adja-

* As the value assumed for r, was derived from this equation, the theoretical and assumed results of course correspond exactly. The bracketed number corresponds to the ordinary value $\frac{d_4}{r_1} = 216.4$. The estimates vary from 214.9 to 217.5.

† According to Johnston's Physical Atlas, the average of the air and ocean temperatures on the parallels of 45° latitude, is 53°.69 F. The specific gravity of air at that temperature is 1 ÷ 807 45.

‡ The retardations of the atmospheric tides at St. Helena, at 0^h, 6^h, 12^h, and 18^h respectively, are 59^m, 85^m, 26^m and 31^m, the mean retardation being $50\frac{1}{4}^m$ [*See Trans. A. P. S.*, vol. 13, p. 128.]

§ Mean of polar (32°) and equatorial (82°) = 57°. Isothermal of 57° = latitude 43°.

cent particles, do not acquire the oscillatory velocity of light until g_1 has acted for $\frac{1}{2}$ the time of *rotation*. Does this indicate successive vibrations in the directions of three co-ordinate axes? And does the tidal action of the planets contribute to the disturbance from which the vibrations originate? The sun-spot theory, and equations 25, 26, and 27, favor such a hypothesis. The proportionality indicated by (1,)

$$u : v_1 : : g_1 t_1^2 : 2 \pi r_1$$

becomes significant, if we consider that any equatorial particle must move through the distance $2 \pi r_1$ before it returns to the same relative position, and that during the entire series of disturbances, through which it passes in the interval, g_1 is exerting an energy, the resultant of which is equivalent to a fall of $g_1 t_1^2$.

BORING RECORDS FROM THE ANTHRACITE BASIN.

By Mr. P. W. Sheafer.

Record of Lower Boring. Nassau Shaft. One mile north of Scranton.

From Surface below R. Road.	8′ 3″		
Rock,	3′ 0″		11′ 3″
Coal,			1′ 0″
Rock,	1′ 2″		
Sandy Gravel,	4′ 7″		
Slate,	6″		
Rock,	1′ 2″		
Slate,	20′ 3″		
Sand Stone,	8′ 4″		
Light Slate,	4′ 6″		
Dark Slate,	4″		39′ 10″
Coal,			2′ 6″
Dark Slate,	7′ 4″		
Hard Rock,	6″		
Dark Slate,	2′ 1″		9′ 11″
Coal,		3′ 0″	
Slate,		3″	
Coal,		1′ 0″	4′ 3″
Slate,	2′ 1″		
Slate, (hard bands,)	1′ 11″		
Hard Rock,	3′ 6″		
Slate, (hard bands,)	20′ 8″		
Dark Hard Rock,	7′ 1″		
Dark Slate,	10′ 6″		44′ 9″
Coal, pure,		8′ 4″	
Coal, bony,		3″	
Coal, good,		6″	
Coal, bony,		6″	
Coal, good,		11″	10′ 6″
Hard Rock,	1′ 4″		
			128′ 7″